Comment on Mohrhoff's "What Quantum Mechanics is Trying to Tell Us"

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ABSTRACT. Mohrhoff proposes using the Aharonov-Bergmann-Lebowitz (ABL) rule for time-symmetric "objective" (meaning non-epistemic) probabilities corresponding to the possible outcomes of not-actually-performed measurements between specified pre- and post-selection measurement outcomes. It is emphasized that the ABL rule was formulated on the assumption that such intervening measurements are actually made and that it does not necessarily apply to counterfactual situations. The exact nature of the application of the ABL rule considered by Mohrhoff is made explicit and is shown to fall short of his stated counterfactual claim.

Ulrich Mohrhoff[1] distinguishes between "subjective" and "objective" probabilities in quantum mechanics. This type of distinction has been made by others[2]; Mohrhoff proposes to modify it by adding a time-symmetric aspect in which the relevant "facts"[3] describing a quantum system include not just the pre-selection outcome but also the post-selection outcome. However, his discussion of the Aharonov-Bergmann-Lebowitz ("ABL") rule[4] as related to counterfactual statements about possible intervening measurements between pre- and post-selection does not address the numerous objections in the literature to the counterfactual usage of ABL

1

probabilities.[5] [6] Moreover, his usage of the ABL rule implicitly assumes special conditions. When these special conditions are made explicit, it appears that the actual claim being made is distinct from, and arguably weaker than, the claim as stated in [1].

The ABL rule was formulated on the assumption that intervening measurements at a time t, $t_a < t < t_b$, are actually made, and would therefore appear to correspond to what Mohrhoff is calling "subjective" probabilities. However, he wishes to consider time-symmetric *objective* probabilities in which a possible intervening measurement of some observable Q is *not* actually performed. Various proofs have been given of the inapplicability of the ABL rule to this kind of situation. Those proofs demonstrated that the counterfactual usage of the ABL rule yields consequences that are inconsistent with quantum theory (see footnote [5], esp. Sharp & Shanks 1993, Cohen 1995, Miller 1996). Vaidman has questioned the validity of such proofs[7] and they have been defended in Cohen [5, 1998] and Kastner [5][8]. Thus there is still an active controversy on this point which Mohrhoff does not address in [1].[9]

Mohrhoff claims that the ABL rule can be applied to statements such as:

Statement 1: "If a measurement of observable Q were performed on system S between the (actual) preparation of the probability measure $|a\rangle\langle a|$ at time t_a and the (actual) observation of the property $|b\rangle\langle b|$ at time t_b, but no measurement is actually performed between t_a and t_b, then the measurement of Q would yield q_i with probability $p(q_i|a,b)$."[10]

Now, if we look at Mohrhoff's application of the ABL rule, what is actually being done is the following. A possible world j is found in which system S happens to have the same pre- and post-selection outcomes as it did in the actual world (i). The ABL rule is then applied, not to the actual world, but to this possible world j.[11] Thus Mohrhoff's application actually supports

the following statement, which differs significantly from Statement (1):

Statement 2: "In the possible world j in which observable Q is measured and system S yields outcomes a and b at times t_a and t_b respectively, the probability of obtaining result q_k at time t is given by $P_{ABL}(q_k|a,b)$."

It should be noted that the possible world j referred to in Statement 2—the one in which system S happens to end up with the same pre- and post-selection results as in the actual world— is only one member of a set W of worlds which differ from the actual world in that observable Q *is* measured. The other worlds in this set are worlds in which observable Q is measured at time t and the pre- and/or post-selection outcomes are *not* the same as in the actual world.

To sum up the situation so far: In Mohrhoff's counterfactual claim, the ABL rule is applied not to the actual world (in which no measurement was performed at time t), but rather to a possible world in which the measurement at time t is performed and the quantum system has the same pre- and post-selection outcomes as in the actual world. The result thus obtained is claimed to apply to the actual world inasmuch as it is claimed to give an answer to a question about how our world might have been different between the times t_a and t_b (with their associated outcomes), were a certain measurement made (i.e., it is claimed to provide a basis for the truth of Statement 1). Let us call this kind of claim a "counterfactual†," where the "†" signifies that it is a new type of counterfactual claim which may or may not be immune to the type of objections previously raised in the literature. Our task, then, is to analyse this counterfactual† and determine whether it avoids the conclusion of proofs such as that of Sharp & Shanks [5].[12]

Let us first clarify what is involved in considering hypothetical situations involving measurements that were not actually made. Consider the following question from the viewpoint of an experimenter— a physicist to whom we

will henceforth refer as "Dr. X"— who preselects particles in state $|a\rangle$ at t_a and post-selects particles in state $|b\rangle$ at t_b.

Dr. X asks himself: "In general, what would have happened if I had made a measurement at time t that I did not, in fact make? How might the data of my experiment change?"

One part of the answer to this question is that particles that were found to be in state $|b\rangle$ at t_b might not have ended up in the same state. [13]

Therefore, from Dr. X's point of view, the appropriate and correct counterfactual statement of the ABL rule would be worded as follows:

Statement 1′: "Consider system S having pre- and post-selection results a and b at times t_a and t_b when a measurement of observable Q was not performed. If a measurement of observable Q had been performed at time $t, t_a < t < t_b$ on S, *and if S had the same pre- and post-selection outcomes as above*, outcome q_k would have resulted with probability $P_{ABL}(q_k|a,b)$,"

Now, for Dr. X the second, italicized "if" clause is a big "if," in view of his question and answer above. Acknowledging this second "if" takes into account that the background conditions that must hold in order to apply the ABL rule counterfactually to system S—namely, that it must have the same pre- and post-selection results as in the actual world—are not guaranteed to hold if the measurement is actually performed. (See footnote [13].)

In the absence of the caveat of the second "if," which considerably weakens the counterfactual claim, such background conditions have to satisfy the following requirement, which is itself a counterfactual statement:

Requirement C: If a measurement of observable Q had been performed, system S would (with certainty) have been pre- and post-selected with outcomes a and b as in the actual world.

This requirement is often referred to as "cotenability": if the necessary

background conditions are not *cotenable* with the antecedent (i.e., the measurement of Q), then a counterfactual statement such as Statement 1, which crucially depends on the stability of those background conditions, fails.

Now, it is obvious that in the pre- and post-selection situation considered by Mohrhoff, Requirement C is not satisfied. As Dr. X observes above, if he had in fact measured Q, then system S might not have been post-selected. However, Mohrhoff argues that cotenability is not an issue for his new counterfactual† ; all he requires is that a possible world j exists in which he can apply the ABL rule, with pre- and post-selection results a and b, to system S. Therefore his argument is essentially the following: to make a counterfactual claim like that of Statement (1), all one need do is to find a possible world in which required background conditions happen to hold, apply the ABL rule to the counterpart of system S in that possible world, and then claim that the result applies to the actual system S.

A diagram may help to make clear the exact nature of Mohrhoff's counterfactual† claim.[14] It is argued that one should take into account that the outcome of the final measurement was the value b. This outcome should be viewed as "fixed," the idea being that the final conditions should have the same status as the initial conditions; in other words, one assumes a two-valued temporal boundary condition. This is equivalent to assuming that the appropriate probability to be assigned to the event b at t_2 is unity, the *posterior* probability of outcome b (since it actually happened). That is, questions about what might have happened other than outcome b, are now viewed as irrelevant, since the applicable probabilities for such questions are *prior* probabilities and as such do not take into account all relevant facts.

Now, one considers a system of possible worlds with the following characteristics (refer to Figure 1):
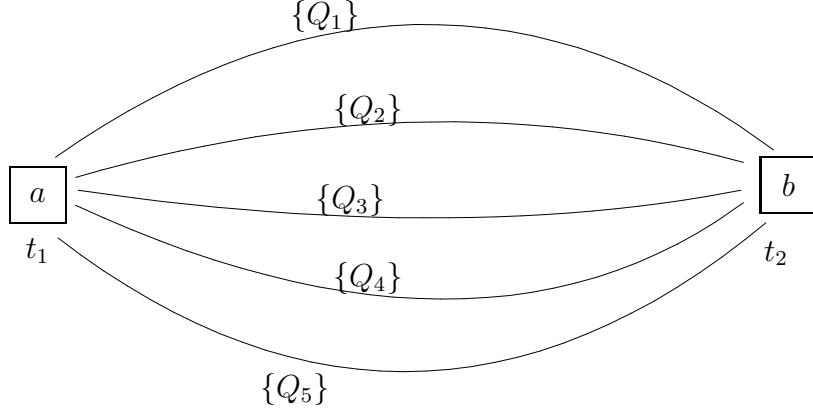
Figure 1. The system of possible worlds assumed in Mohrhoff's ABL counterfactual†.

Each of the curved lines represents a set of possible worlds $\{Q_j\}$ in which a measurement of the observable Q_j is performed at time t. The individual member worlds of each set $\{Q_j\}$ are identified with the possible outcomes of measurements of Q_j. (Thus a particular set of worlds $\{Q_j\}$ here corresponds to the possible world j referred to above, with additional structure.) Within this proposed construct, the ABL probability is assumed to give the *subjective* probability applying to an observer Dr. X† associated with the set of worlds $\{Q_j\}$ in which the measurement of Q_j is performed.[15] In Mohrhoff's counterfactual† , this probability is then held to apply (objectively) to a different world—the world of Dr. X—in that it is claimed to give an answer to the question corresponding to Statement (1), i.e.: "If (contrary to fact) a measurement of Q_j had been performed between the

actual outcomes at t_a and t_b, what would be the probability that Dr. X would find outcome q_{j_k} (one of the eigenvalues of Q_j)?"

However, such a claim would seem to conflate two distinctly different perspectives or frames of reference. In being invoked in Statement (1), the ABL rule is being applied counterfactually to Dr. X's world, since it is from that frame of reference that the claim is being made. Yet when called upon to justify his counterfactual† , Mohrhoff argues from the non-counterfactual perspective of Dr. X† .

For example, Mohrhoff's objection to the Sharp & Shanks proof, like Vaidman's, consists in demanding that the "counterfactual" measurement be considered as performed, in contrast to the proof which assumes that, in accordance with Dr. X's perspective, the measurement is *not* performed. Thus, according to Mohrhoff, the calculation should be considered as applying to the world(s) of Dr. X† rather than to the world of Dr. X. But Dr. X† 's perspective is markedly different from that of Dr. X, and it is Dr. X who is making the counterfactual claim (Statement 1). The proof applies to Dr. X and his claim, not to someone who has actually performed the measurement. One does not refute a proof by arguing that it does not apply to a frame of reference for which it was not intended. Thus Mohrhoff's counterfactual† formulation, which postulates a hypothetical system of worlds in which *non*-counterfactual applications of the ABL rule are held to apply to those possible worlds, fails to evade the conclusion of the proofs which address the *actual* world in which no such measurement is performed.

To conclude, it has been argued that Mohrhoff's application of the ABL rule, which depends on a specially chosen system of possible worlds, fails to support his stated counterfactual claim based on that rule. Moreover, his objection to the proofs demonstrating the nonvalidity of the counterfactual usage of the ABL rule fails to refute those proofs, because it does not apply to the actual world addressed by the proofs.

Finally, it should be noted that none of the arguments in this Comment presuppose any time-asymmetry assumptions. Nor are any underlying metaphysical assumptions about time "flow" or subjective, "classical" experience invoked or required. The present author fully agrees with the basic result (relevant to this discussion) originally obtained by ABL, which was the following: "...during the time interval between two noncommuting observations, we may assign to a system the quantum state corresponding to the observation that follows with as much justification as we assign, ordinarily, the state corresponding to the preceding measurement."[16]

The above quoted statement considers *either* temporal direction as being equally valid as far as quantum theory is concerned (where here, "temporal direction" simply means whether one regards the parameter t as increasing or decreasing in the applicable laws). However, it does not consider *combining both temporal directions* as is implied in a counterfactual usage of the ABL rule. I.e., nothing in the ABL paper suggests holding fixed *both* pre- and post-selection states while considering not-actually -performed measurements during that time interval.[17]

¹U. Mohrhoff. 'What Quantum Mechanics is Trying to Tell Us,' *American Journal of Physics 68*, 728-745 (2000).

²See, for example, R.I.G. Hughes. *The Structure and Interpretation of Quantum Mechanics* (Harvard University Press, Cambridge, MA, 1989, p. 218.; A. Shimony. "Search for a worldview which can accomodate our knowledge of microphysics," in J. Cushing and E. McMullin, eds., *Philosophical Consequences of Quantum Theory* (University of Notre Dame Press, Notre Dame, 1989), p. 27; A. Shimony, *Search for a Naturatistic World View, Vol. II* (Cambridge University Press, New York, NY, 1993), pp. 141-2. Mohrhoff defines subjective probabilities as applying only in cases in which measurements have been made and an observer is ignorant of the result of the measurement, which differs slightly from Hughes' use of the term (see note [3]).

³Mohrhoff applies the term "fact" to measurement outcomes only (whether known or unknown), as opposed to possessed properties independent of measurement (which he denies).

⁴Y. Aharonov, P. G. Bergmann, and J. L. Lebowitz. 'Time Symmetry in the Quantum Process of Measurement,' *Physical Review B 134*, 1410-16 (1964).

⁵See, for instance, J. Bub and H. Brown. 'Curious Properties of Quantum Ensembles Which Have Been Both Preselected and Post-Selected,' *Physical Review Letters 56*, 2337-2340 (1986); W. Sharp and N. Shanks. 'The Rise and Fall of Time-Symmetrized Quantum Mechanics,' *Philosophy of Science 60*, 488-499 (1993); O. Cohen. 'Pre- and postselected quantum systems, counterfactual measurements, and consistent histories, ' *Physical Review A 51*, 4373-4380 (1995); 'Reply to "Validity of the Aharonov-Bergmann-Lebowitz Rule", ' *Physical Review A 57*, 2254-2255 (1998); D. J. Miller. 'Realism and Time Symmetry in Quantum Mechanics,' *Physics Letters A 222*, 31-36 (1996); R. E. Kastner. 'Time-Symmetrised Quantum Theory, Counterfactuals, and "Advanced Action,"' *Studies in History and Philosophy of Modern Physics 30*, 237-259 (1999); 'The Three-Box Paradox and Other Reasons to Reject the Counterfactual Usage of the ABL Rule,' *Foundations of Physics 29*, 851-863 (1999); 'TSQT "Elements of Possibility"?,' *Studies in History and Philosophy of Modern Physics 30*, 399-402 (1999).

⁶As it happens, Mohrhoff's specific example of a counterfactual use of the ABL rule corresponds to a special case in which that use is valid (in the strong sense of Statement 1). This is an example in which a particle is pre- and post-selected with outcomes a and b corresponding to noncommuting observables A and B, and counterfactual measurements of either A or B are considered at time t. (The validity of a counterfactual usage of the ABL rule in cases like this has been shown in detail in Kastner [5] and in Cohen [5].) But this is a special case and, as has been discussed at length in the literature, "would"-type counterfactual uses of the ABL rule are generally invalid.

⁷L. Vaidman. 'Validity of the Aharonov-Bergmann-Lebowitz Rule,' *Physical Review A 57*, 2251-2253 (1998).

⁸In his reply to this Comment, Mohrhoff rejects my defense of the Sharp and Shanks proof on the basis that I allegedly assume the reality of ensembles in situations concerning only one particle. This remark misunderstands my use of ensembles. The term "ensemble" as I am using it denotes not a real collection of particles, but rather a *conceptual* ensemble in the sense of statistical mechanics (cf. R. K. Pathria. *Statistical Mechanics* (Pergamon Press, Oxford, 1972), p. 4).

⁹In his reply to this Comment, Mohrhoff essentially repeats the objection to the Sharp and Shanks proof previously given by Vaidman[7]. In this attempt to refute the proofs of Sharp and Shanks and others (which all have essentially the same structure), Vaidman and Mohrhoff simply assume that the "counterfactual" measurement *is performed* in all expressions employed in the proof. Then, of course, there can be no inconsistency with the

predictions of quantum mechanics. But this is no refutation, for it explicitly *assumes as true that which is manifestly false*: namely, that the "counterfactual" measurement is actually made. This procedure is then justified by claiming that the computation applies not to the actual world but to a specially chosen possible world. If such a procedure were to be allowed, then one could argue for something manifestly false in the actual world merely by finding a specially chosen possible world in which it is true.

[10]'Objective probabilities, quantum counterfactuals, and the ABL rule: Apropos of Kastner's comment,' quant-ph/0006116 (2000) (a preprint version of Mohrhoff's reply to the present Comment).

[11]U. Mohrhoff, quant-ph/0006116, p. 4: "If we think of the measurement of Q as taking place in a possible world, we consider a world in which all the relevant facts are exactly as they are in the actual world, except that in this possible world there is one additional relevant fact indicating the value possessed by Q at a time between t_a and t_b."

[12]Sharp and Shanks (1993) consider an ensemble of spin-$\frac{1}{2}$ particles prepared at time t_1 in the state $|a_1\rangle$ (read as 'spin up along direction $\mathbf{a}$'). They then assume that this ensemble is subjected to a final post- selection spin measurement at time t_2 along direction $\mathbf{b}$ (i.e., the observable σ_b is measured). This measurement yields two subensembles $E_i, i = 1, 2$ corresponding to results spin up or spin down along direction $\mathbf{b}$. The weight of each subensemble E_i is given by $|\langle b_i|a_1\rangle|^2$.

Now they consider each subensemble individually, asking the counterfactual question: If we had measured the spin of these particles along direction $\mathbf{c}$ (i.e., observable σ_c) at a time t between t_1 and t_2, what would have been the probability for outcome c_1? They use the ABL rule to calculate the probability of outcome c_1 for each subensemble E_i for such a counterfactual measurement. They then show that the total probability of outcome c_1 derived from the above calculation, taking into account the weights of the two subensembles E_i, in general disagrees with the quantum mechanical probability, which is given simply by $|\langle c_i|a_1\rangle|^2$.

[13]One can, of course, deny this statement if one assumes fatalism (i.e., everything that happens *must* happen). But then it must also be assumed that there is no possibility of a "counterfactual" measurement at time t, since it is a recordable matter of fact that no such measurement occurred, and according to fatalism, that documented absence of a measurement is also a fact that *must* happen.

[14]Mohrhoff has confirmed in a private correspondence that the diagram discussed herein correctly illustrates his proposed possible world structure.

[15]However, this assumption can be disputed, since under the given construct (which assumes that outcome b definitely occurs at t_2), the conditional probabilities $P(b|q_{j_k})$ (where q_{j_k} is an eigenvalue of the associated observable Q_j) are unity, rather than the standard quantum mechanical conditional probabilities as assumed in the ABL rule.

[16]ABL (1964), abstract.

[17]ABL (1964) do say "We shall now consider an ensemble of systems whose initial and final states are fixed to correspond to the particular eigenvalues a and b, respectively; we ask for the probability that the outcome of the intervening measurements are $d_j, ...d_n$, respectively." (p. B1412) But those outcomes correspond to *actually performed* measurements.